Understanding Interaction Models: Improving Empirical Analyses

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- What's a conditional hypothesis?

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- Arguably, any causal story implies a set of conditions that needs to be satisfied before a purported cause is sufficient to bring about its effect.

Conditional Hypotheses

Despite the ubiquity of conditional hypotheses, we find that the execution of interaction models is often flawed and inferential errors are common.

Four Recommendations

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1. Include interaction terms.
2. Include all constitutive terms.
3. Do not interpret constitutive terms as unconditional marginal effects.
4. Calculate substantively meaningful marginal effects and standard errors.

Include Interaction Terms

Recommendation 1: Include interaction terms when you have a conditional hypothesis.

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When Z = 1, we have

$$Y = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X + \epsilon$$

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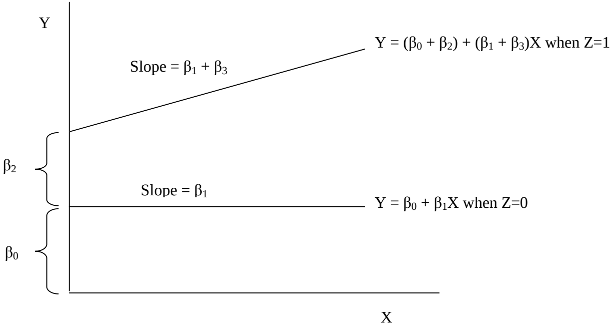
Hypothesis H_1: An Increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent



Include All Constitutive Terms

Recommendation 2: Include all constitutive terms when specifying multiplicative interaction models except in very rare circumstances.

- Constitutive terms are those elements that constitute the interaction term.

Interaction Term	Constitutive Terms
XZ	X, Z
X^2	X
XZJ	X, Z, J, XZ, XJ, ZJ

Include All Constitutive Terms

What happens if we omit Z from our model specification?

$$Y = \gamma_0 + \gamma_1 X + \gamma_3 XZ + \nu$$

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Common justifications for excluding constitutive terms are:

1. "I believe that Z does not have any effect on Y on average and so can be excluded."
2. "I believe that Z does not have any effect on Y when X is zero and so can be excluded."

Both justifications are based on the expectation that $\beta_2 = 0$.

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This justification is plain wrong because β_2 does not represent the average effect of Z on Y; it only indicates the effect of Z on Y when $X = 0$.

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There is reason to believe that omitting a constitutive term may still lead to inferential errors even when the analyst has a strong conditional theory like this.

The basic reason is that the analyst's theory might be wrong and β_2 might not equal 0. Presumably we want to test our theory and not assume that it is correct.

Include All Constitutive Terms: Omitted Variable Bias

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If $\beta_2 \neq 0$ and Z is correlated with either XZ (or X), then omitting Z will result in biased estimates of β_0, β_1, and β_3.

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

Include All Constitutive Terms: Omitted Variable Bias

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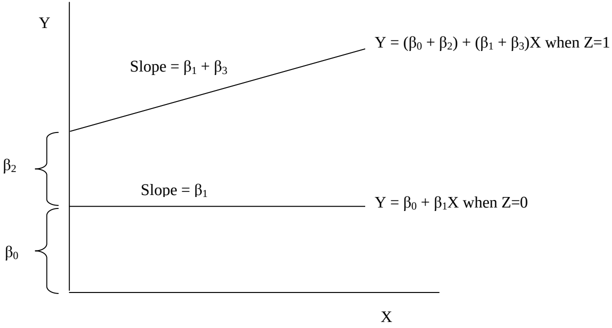
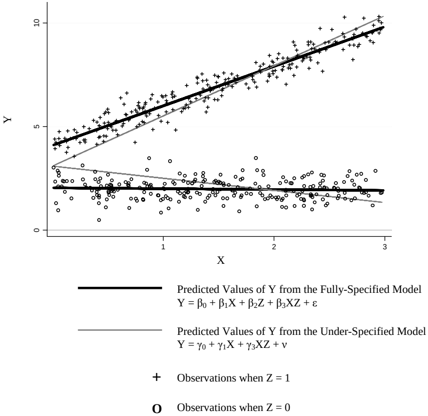


Include All Constitutive Terms: Omitted Variable Bias

Figure: Scatter Plot of 500 Observations: $\beta_0 = 2, \beta_1 = 0, \beta_2 = 2, \beta_3 = 2$



Include All Constitutive Terms: Omitted Variable Bias

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

$$Y = \gamma_0 + \gamma_1 X + \gamma_3 XZ + \nu$$

How much bias?

Include All Constitutive Terms: Omitted Variable Bias

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

$$Y = \gamma_0 + \gamma_1 X + \gamma_3 XZ + \nu$$

How much bias?

- $\gamma_0 = \beta_0 + \beta_2 \alpha_0$
- $\gamma_1 = \beta_1 + \beta_2 \alpha_1$
- $\gamma_3 = \beta_3 + \beta_2 \alpha_3$

where the αs are the coefficients from the regression of Z on X and XZ i.e.

$$Z = \alpha_0 + \alpha_1 X + \alpha_3 XZ + \varepsilon$$

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Possibly, but at least two necessary conditions must be met before an analyst considers omitting it.

1. The analyst must have a strong theoretical expectation that Z has no effect on Y when X is 0 i.e. $\beta_2 = 0$.
2. The analyst should estimate the fully-specified model and find that $\beta_2 = 0$.

Include All Constitutive Terms: Condition 1

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The only situation in which this theoretical expectation is justified a priori is if X is measured with a natural zero.

- The main reason for this is that we can rescale our variables to make the coefficients on the constitutive terms anything we want them to be.

Include All Constitutive Terms: Condition 1

Imagine that we add some arbitrary constant L to X to create X^*.

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$$Y = \beta_0 + \beta_1(X^* - L) + \beta_2 Z + \beta_3(X^* - L)Z + \epsilon$$

Rewriting, we get

$$Y = (\beta_0 - \beta_1 L) + \beta_1 X^* + (\beta_2 - \beta_3 L)Z + \beta_3 X^* Z + \epsilon$$

It should be clear that rescaling X in this arbitrary way changes the coefficient on Z from β_2 to $\beta_2 - \beta_3 L$. The standard error of the coefficient on Z also changes.

Include All Constitutive Terms: Condition 1

$$Y = (\beta_0 - \beta_1 L) + \beta_1 X^* + (\beta_2 - \beta_3 L)Z + \beta_3 X^* Z + \epsilon$$

The problem: β_2 may truly be zero, but we have no way of knowing in practice if we are estimating β_2 or $\beta_2 - \beta_3 L$ if our theory does not tell us which particular scale to use for X.

- The analyst has no way of predicting a priori what the coefficient on Z will be before estimating her model.

Include All Constitutive Terms: Condition 2

The analyst should estimate the fully-specified model and find that $\beta_2 = 0$.

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Note that even if β_2 is statistically indistinguishable from zero, the other parameters of interest will still be estimated with bias to the extent that β_2 is not exactly zero if the constitutive term is dropped.

- Much depends on whether β_2 is close to zero and whether β_2 is small relative to β_1 and β_0.

Include All Constitutive Terms: Always?

There are a limited set of circumstances in which omitting a constitutive term would not lead to significant inferential errors:

1. The analyst must have a strong theoretical expectation that Z has no effect on Y when X is 0 i.e. $\beta_2 = 0$.
2. The analyst should estimate the fully-specified model and find that $\beta_2 = 0$.

Might as well report the results from the fully-specified model!

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It is true that including constitutive terms can increase multicollinearity, but this does **not** justify omitting them.

Include All Constitutive Terms: Multicollinearity

Problems with multicollinearity are often overstated.

Scholars often worry about multicollinearity when they see that the coefficients from a linear-additive model change when an interaction term is included in the model.

- In the linear-additive world, the sensitivity of results to the inclusion of a variable is often taken as a sign of multicollinearity.

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- In the linear-additive world, the sensitivity of results to the inclusion of a variable is often taken as a sign of multicollinearity.

This need not be the case with interaction models, though.

- The coefficients in linear-additive and interaction models capture different quantities and so they will almost certainly differ irrespective of whether there is multicollinearity or not.

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Typically, the analyst is interested in the marginal effect of X.

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

$$\hat{\sigma}_{\frac{\partial Y}{\partial X}} = \sqrt{var(\hat{\beta}_1) \ + \ Z^2 var(\hat{\beta}_3) \ + \ 2Zcov(\hat{\beta}_1 \hat{\beta}_3)}$$

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This is **NOT** true.

Basic intuition: Multicollinearity issues arise due to a lack of information in the data. Centering does not add information. Therefore, centering does not help.

Include All Constitutive Terms: Multicollinearity

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

$$Y = \delta_0 + \delta_1 X_c + \delta_2 Z_c + \delta_3 X_c Z_c + \epsilon_c$$

where the variables have been centered by subtracting their means i.e. $X_c = X - \overline{X}$ and $Z_c = Z - \overline{Z}$.

With some rewriting, we have:

$$\begin{aligned} Y &= \delta_0 - \delta_1 \overline{X} - \delta_2 \overline{Z} + \delta_3 \overline{X}\,\overline{Z} \\ &+ (\delta_1 - \delta_3 \overline{Z})X + (\delta_2 - \delta_3 \overline{X})Z + \delta_3 XZ + \epsilon_c \end{aligned}$$

Include All Constitutive Terms: Summary

Bottom Line: While the omission of constitutive terms may well reduce multicollinearity, it is probably unwise, almost always unnecessary, and can be justified only in extremely rare circumstances.

Interpretation of Constitutive Terms

Recommendation 3: Do not interpret constitutive terms as unconditional marginal effects.

Interpretation of Constitutive Terms

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Interactive world:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

Interpretation of Constitutive Terms

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

β_1 only captures the marginal effect of X on Y when Z=0.

Substantive Marginal Effects and Standard Errors

Recommendation 4: Calculate substantively meaningful marginal effects and standard errors.

Substantive Marginal Effects and Standard Errors

A typical table of results does not normally convey the necessary information to test a conditional hypothesis.

1. We only know the effect of X on Y when Z=0.
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1. We only know the effect of X on Y when Z=0.
2. It is not always possible to know if X has a meaningful conditional effect on Y from simply looking at the magnitude and significance of the coefficient on the interaction term.

It is nearly always the case that the analyst should go beyond the traditional table of results in order to convey quantities of interest such as the marginal effect of X on Y across different values of Z.

Substantive Marginal Effects and Standard Errors

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Interaction Term	$Z = 0$	$Z = 1$
Marginal Effect of X	β_1	$\beta_1 + \beta_3$
Standard Error	$\sqrt{var(\hat{\beta}_1)}$	$\sqrt{var(\hat{\beta}_1) \; + \; var(\hat{\beta}_3) \; + \; 2cov(\hat{\beta}_1 \hat{\beta}_3)}$

Substantive Marginal Effects and Standard Errors

If your modifying variable Z is continuous, you can provide the appropriate quantities of interest in the form of a figure.

Let's see an example from Golder (2006).

Substantive Marginal Effects and Standard Errors

Hypothesis: Temporally-proximate presidential elections will reduce the effective number of legislative parties if and only if the number of presidential candidates is sufficiently low.

$$\begin{aligned} ElectoralParties = \beta_0 &+ \beta_1 Proximity + \beta_2 PresidentialCandidates \\ &+ \beta_3 Proximity * PresidentialCandidates \\ &+ \beta_4 Controls + \epsilon \end{aligned}$$

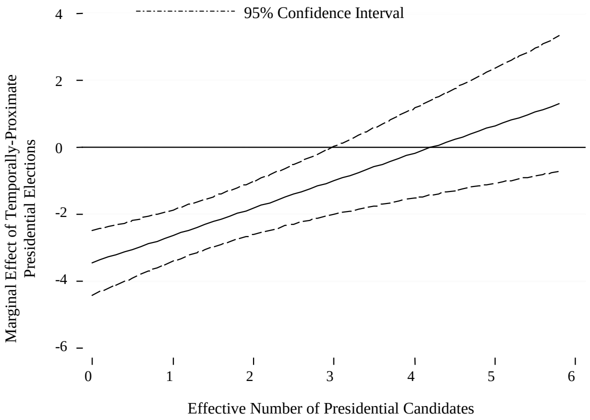
Substantive Marginal Effects and Standard Errors

Dependent Variable: Effective Number of Electoral Parties

Regressor	Model
Proximity	-3.44**
	(0.49)
PresidentialCandidates	0.29*
	(0.07)
Proximity*PresidentialCandidates	0.82**
	(0.22)
Controls	——
Constant	3.01**
	(0.33)
R^2	0.34
N	522

* $p < 0.05$; ** $p < 0.01$ (two-tailed). Control variables not shown here. Robust standard errors clustered by country in parentheses.

Substantive Marginal Effects and Standard Errors



Survey of the Literature

Survey of Articles with Interaction Models (APSR, AJPS, JOP 1998-2002)

Recommendation	Yes	No	Total
Include all Constitutive Terms	107 (69%)	49 (31%)	156
Interpret Constitutive Terms Correctly*	38 (38%)	63 (62%)	101
Provide Range for Marginal Effect	86 (55%)	70 (45%)	156
Provide Measure of Uncertainty	34 (22%)	122 (78%)	156

* Only 101 articles interpreted constitutive terms.

Survey of the Literature

Only 16 of the 156 articles (a) included all constitutive terms, (b) did not make mistakes interpreting this terms, and (c) calculated substantively meaningful marginal effects and standard errors.

Logit and Probit

All of the points that I have made so far regarding the specification and interpretation of interaction models hold when there is a dichotomous, instead of a continuous, dependent variable.

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2. Include all constitutive terms.
3. Do not interpret constitutive terms as unconditional marginal effects.
4. Calculate substantively meaningful marginal effects and standard errors.

There are some ‘complicating’ features, though.

Logit and Probit: To Include or Not to Include?

Some people argue that you do not need to include interaction terms in order to take account of conditional hypotheses since the non-linearity of these models force the effect of the independent variables to depend on each other anyway.

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Additive probit model:

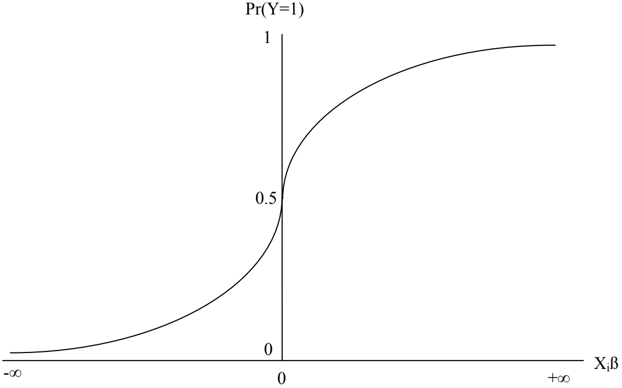
$$Pr(Y = 1) = \Phi(\gamma_0 + \gamma_1 X + \gamma_2 Z) = \Phi(\cdot)$$

The marginal effect of X on Pr(Y=1) is:

$$\frac{\partial \Phi(\cdot)}{\partial X} = \phi(\cdot)\gamma_1$$

Logit and Probit: To Include or Not to Include?

Compression effects are substantively meaningful and can be interpreted as such.



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Typically, when we think of interaction effects in non-linear models, we have a specific hypothesis that the effect of X on Pr(Y=1) depends on the value of some other variable Z (above and beyond compression effects that are always there).

If this is the case, then you should include an interaction term.

Logit and Probit: To Include or Not to Include?

Interactive probit model

$$Pr(Y = 1) = \Phi(\beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ) = \Phi(\cdot)$$

The marginal effect of X on Pr(Y=1) is:

$$\frac{\partial \Phi(\cdot)}{\partial X} = \phi(\cdot)(\beta_1 + \beta_3 Z)$$

Logit and Probit: Interpretation Issues

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This is a big deal and hardly anyone knows this.

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This is a big deal and hardly anyone knows this.

By an "interaction effect", we mean how Z modifies the effect of X on Y i.e. $\frac{\partial^2 Y}{\partial X \partial Z}$.

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In the OLS world, our model is:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

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The marginal effect of X is:

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

And the interaction effect or modifying effect of Z is:

$$\frac{\partial^2 Y}{\partial X \partial Z} = \beta_3$$

Logit and Probit: Interpretation Issues

In the logit world, our model is:

$$P(Y=1) = \frac{1}{1+e^{-x_i\beta}} = \Lambda(\beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ) = \Lambda$$

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The marginal effect of X on Pr(Y=1) is:

$$\frac{\partial P(Y=1)}{\partial X} = [\Lambda(1-\Lambda)][\beta_1 + \beta_3 Z]$$

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The marginal effect of X on Pr(Y=1) is:

$$\frac{\partial P(Y=1)}{\partial X} = [\Lambda(1-\Lambda)][\beta_1 + \beta_3 Z]$$

And the interaction effect or modifying effect of Z is:

$$\frac{\partial^2 P(Y=1)}{\partial X \partial Z} = \beta_3\Lambda(1-\Lambda) + (\beta_1 + \beta_3 Z)(\beta_2 + \beta_3 X)\Lambda(1-\Lambda)(1-2\Lambda)$$

Logit and Probit: Interpretation Issues

The bottom line is that you should not draw inferences about the modifying effect of Z (the interaction effect) from the sign and significance of the interaction term i.e. β_3.

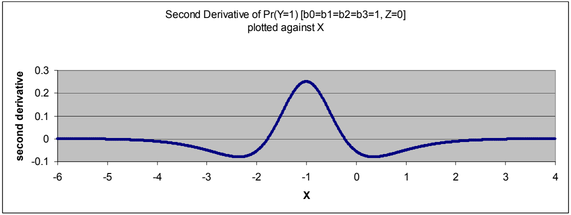
We ran a simulation:

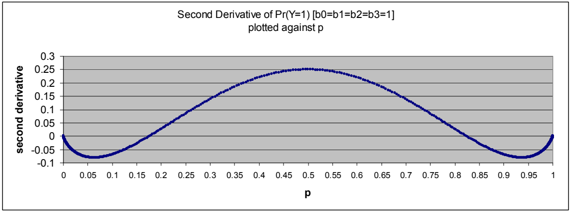
$$x_i\beta = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ \tag{1}$$

and we set the values of the parameters to all be 1 i.e.

$$\beta_0 = \beta_1 = \beta_2 = \beta_3 = 1 \tag{2}$$

Logit and Probit: Interpretation Issues





Substantive Marginal Effects and Standard Errors

Let's look at an example from S. Golder (2006)

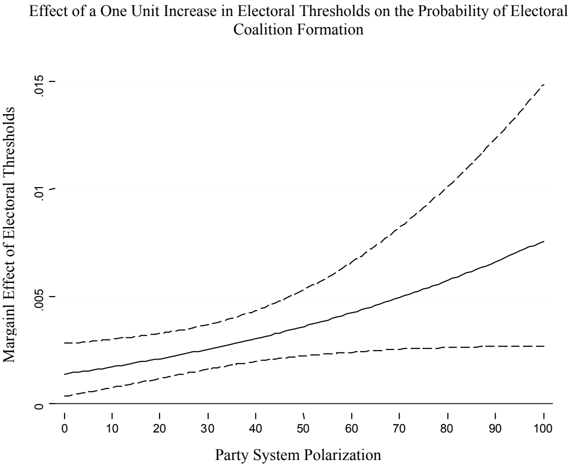
Substantive Marginal Effects and Standard Errors

Let's look at an example from S. Golder (2006)

Hypothesis: An increase in the disproportionality of the electoral system will increase the probability of forming a pre-electoral coalition. This positive effect should be stronger when the party system is polarized.

$$\begin{aligned} PEC^* = \beta_0 &+ \beta_1 Threshold + \beta_2 Polarization \\ &+ \beta_3 Threshold * Polarization \\ &+ \beta_4 Controls + \epsilon \end{aligned}$$

Logit and Probit: Interpretation Issues



Conclusion

Four recommendations

1. Include interaction terms.
2. Include all constitutive terms.
3. Do not interpret constitutive terms as unconditional marginal effects.
4. Calculate substantively meaningful marginal effects and standard errors.